

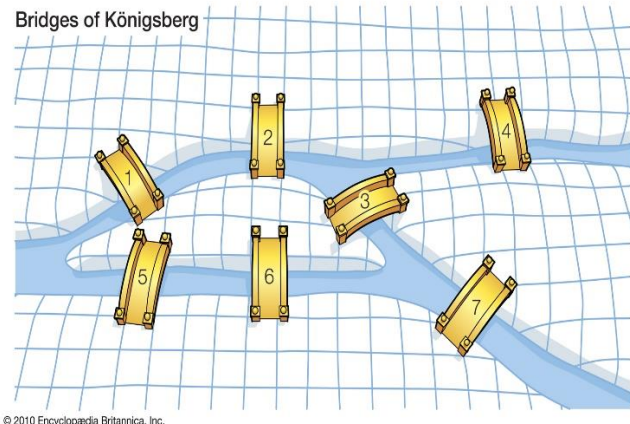
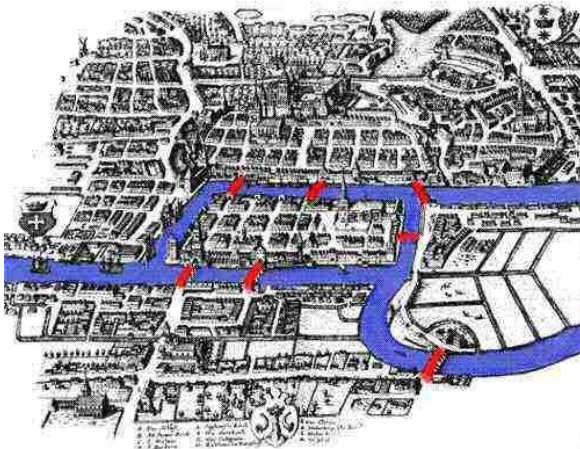


## Connecting Math to Our Lives and Communities

### The Bridges of Königsberg

#### ■ Introduction

- Königsberg was the name for the historic Prussian city that is now Kaliningrad, Russia.
- The city was split by the Pregel river at the centre of the river were 2 small islands.
- The two islands were connected to each other and the riverbank by 7 bridges.
- The problem arises when Carl Gottlieb Ehler was curious to see if he could find a route that let someone cross all 7 bridges without crossing any of them more than once.



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- Can you find a path that uses each bridge once?

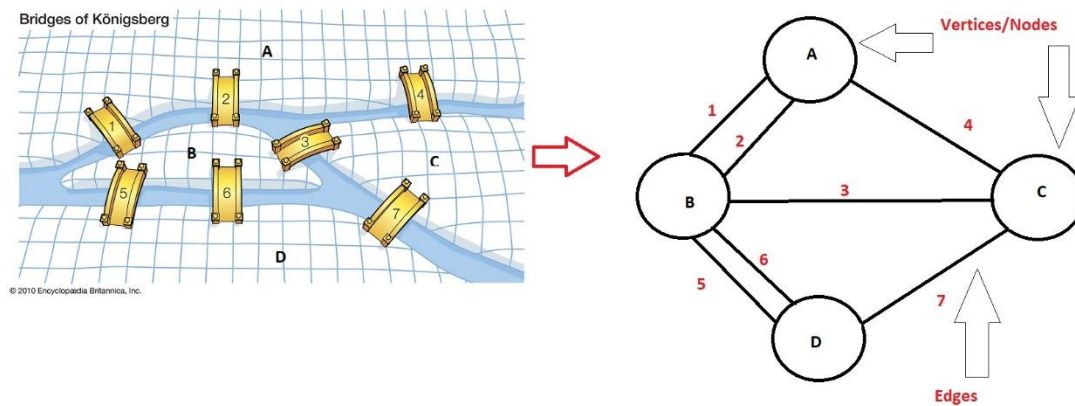
- Math Application

- The question of finding a path through the seven bridges led to a new field of mathematics, called GRAPH THEORY

- Introduction to Graph Theory

- What is graph theory?
- In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices (also called nodes), which are connected by edges.

- Graph Theory Application to Our Problem



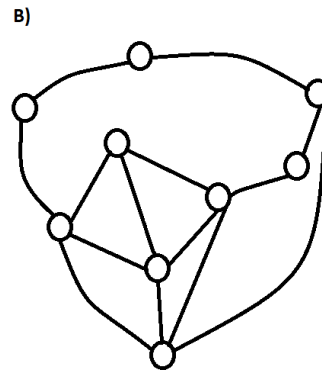
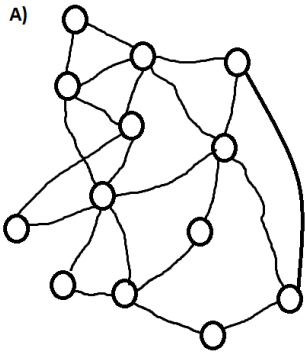
- The land becomes a vertex and the bridges become edges.
- After trying the problem and seeing if there is a path, did you notice that you could not find a path that goes through all seven bridges once.

- Leonhard Euler

- Was Swiss mathematician and physicist, one of the founders of pure mathematics.
- How does he tie into the Bridges of Königsberg problem? The problem was given to Euler by Ehlers. First Euler was sceptical as he did not see there to be a relation to math but after some work, he discovered the field of mathematics called graph theory.

- What Makes a Graph Eulerian?

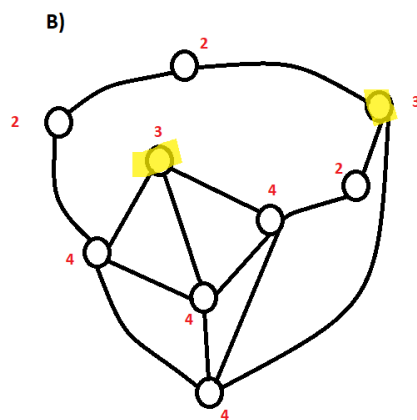
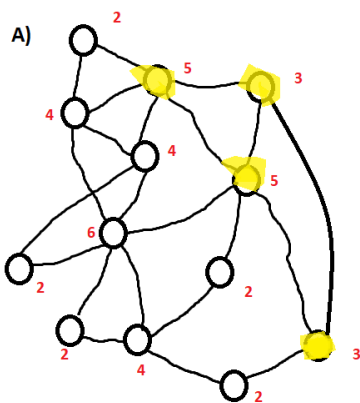
- DEFINITION- An Euler trail in a graph is a trail that includes all the edges of the graph exactly once. A closed Euler Trail is called an Euler Tour. A graph is Eulerian if it has an Euler Tour.
- Have a look at these graphs and see if they are Eulerian.



- Looking at these graphs it seems like its going to be difficult to find if there is path through the graph that uses each edge only once. Lucky for us Euler did some work and found a theorem for us.

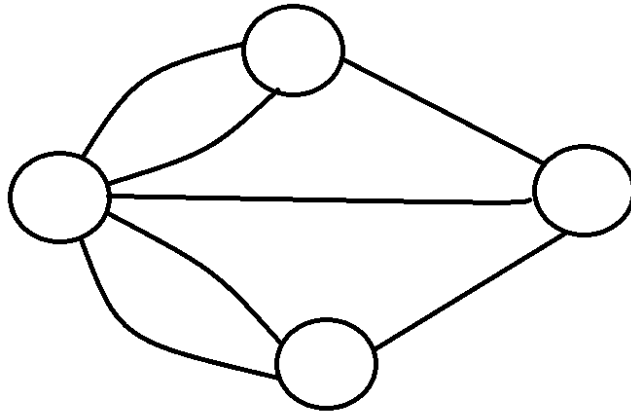
▪ **Theorem**

- A graph is Eulerian if and only if every vertex of the graph has an even degree, there is an exception. A graph can have 2 vertices with odd degree.
- Using this theorem, it makes it much easier to find out if a graph is Eulerian, we just have to count the number of edges leaving a vertex and determine how many odd degree vertices there are.



- Here we can see that graph A is not Eulerian since it has more than 2 odd-degree vertices. However, B is Eulerian since there are only 2 odd-degree vertices.

- Back to The Seven Bridges



- Is this graph Eulerian?